

Question 1

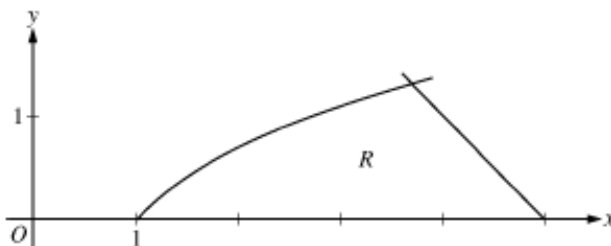
t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

Question 2

Let R be the region in the first quadrant bounded by the x -axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

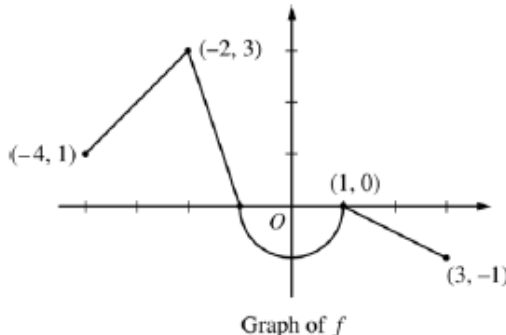


- Find the area of R .
- Region R is the base of a solid. For the solid, each cross section perpendicular to the x -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k .

No Calculator Allowed (60 minutes)

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t) dt$.



- Find the values of $g(2)$ and $g(-2)$.
- For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.
- Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
- For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \leq x \leq 5$.

- (a) Find $f'(x)$.
- (b) Write an equation for the line tangent to the graph of f at $x = -3$.
- (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$
Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.
- (d) Find the value of $\int_0^5 x\sqrt{25 - x^2} \, dx$.

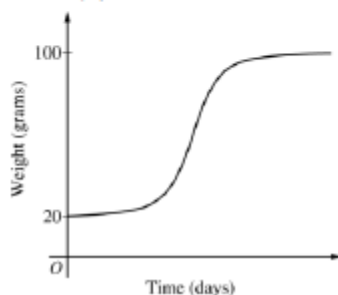
Question 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find $\frac{d^2B}{dt^2}$ in terms of B . Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.
- (c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.



Question 6

For $0 \leq t \leq 12$, a particle moves along the x -axis. The velocity of the particle at time t is given by

$$v(t) = \cos\left(\frac{\pi}{6}t\right). \text{ The particle is at position } x = -2 \text{ at time } t = 0.$$

- (a) For $0 \leq t \leq 12$, when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.
- (c) Find the acceleration of the particle at time t . Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
- (d) Find the position of the particle at time $t = 4$.